The Physics of Juggling

Outcomes:
1. Analyze qualitatively and quantitatively the horizontal and vertical motion of a projectile. (325-6)
2. Analyze natural and technological systems to interpret and explain their structure. (116-7)
3. Distinguish between problems that can be solved by the application of physics-related technologies and those that cannot. (118-8)
4. Compile and display evidence and information, by hand or computer, in a variety of formats, including diagrams, flow charts, tables and graphs. (214-3)
5. Analyze and describe examples where technological solutions were developed based on scientific understanding. (116-4)
6. Define and delimit problems, estimate quantities, and interpret patterns and trends in data, and infer or calculate the relationship among variables. (212-2, 213-4, 214-5)

Introduction

An old riddle tells of a 148-pound man who had to cross a canyon over a bridge that could only support 150 pounds (Beek & Lewbel, 1995). Unfortunately the man was carrying three one-pound cannonballs and only had time for one trip across. The solution to the riddle was that the man would juggle the cannonballs while crossing the bridge. In reality, juggling the balls would not have been much help since catching one of the cannonballs would have exerted a force on the bridge that would have exceeded the weight limit. The poor man would have ended up at the bottom of the canyon! Though not very helpful in this particular case, juggling does have relevance beyond riddles or entertainment.

Beek and Lewbel (1995) suggest the application of juggling in the study of human movement, robotics, and mathematics. Studying the mathematics of juggling became popular in the 1980’s though juggling itself is an ancient tradition dating back to Egypt and Rome. The term “juggling” comes from the Latin “joculare” meaning “to jest”. Before the mid-twentieth century juggling was mainly a part of magic shows (Juggling). Public interest in juggling as a hobby increased after 1948 when the first juggling convention was held in the United States. That interest has persisted over the years as people continue to test physical limits for the number of objects juggled. Currently, the world record for the greatest number of objects juggled is 13 rings, 12 balls or 9 clubs (List of Numbers Juggling Records, 2002). While these numbers may seem impossibly high, they are in fact attainable with the right combination of physical ability and physics knowledge.

Theory

Good jugglers make juggling look so easy that it is difficult to imagine all the physics that comes into play. Gravity has a significant effect on the number of objects juggled. Each ball must be thrown high enough to allow the juggler time to handle the other balls. While throwing higher gives the juggler extra
time, it also increases the risk of error. Juggling ‘low’ on the other hand, requires the juggler to catch and throw quickly, also increasing the risk of error. The need for speed or height will also change dramatically as the number of objects being juggled increases. Beek and Lewbel (1995) assert that learning to juggle three balls can be accomplished in just hours or days. This learning time can increase to weeks or months for four balls, and months or years for five balls.

In the cascade, the hands alternate throwing balls to each other, resulting in a figure eight. In the shower pattern the balls are thrown around in a circle. In the fountain pattern the balls are thrown (and caught) simultaneously with both hands (in sync), or by catching a ball with one hand and throwing one with the other at the same time (out of sync). Despite the identification of these patterns, it is important to note that due to factors like the oscillation of the jugglers’ hands, or individual vision and feel, no two throws or catches are exactly the same.

In an attempt to measure juggling consistency, “dwell ratio” has been defined as the “fraction of time that a hand holds on to a ball between two catches (or throws)” (Beek & Lewbel, 1995, p. 3). A large dwell ratio means that the hand cradles the ball for a longer period of time. This means that the juggler has more time to throw accurately. A small dwell ratio means that the balls have a longer time in the air, which allows the juggler time to make corrections to hand repositioning. Novice jugglers typically like larger dwell ratios while professionals tend towards smaller values because they are more interested in shifting patterns.

A knowledge of projectile motion can give a juggler valuable information on the time available to throw and catch balls in a juggling pattern. Let us consider some numbers for throwing one ball in the air at around 2.0 m/s. The equations for projectile motion can tell us how long the juggler has to catch the ball, how high it will rise and about how far apart to keep the hands. In the following calculations assume that the ball is being thrown from the left hand to the right at an angle of 60° to the horizontal (neglecting air resistance on the ball).

Using trigonometry we see that,

\[ v_{x1} = v_1 \cos 60° \]
\[ v_{y1} = v_1 \sin 60° \]

At the peak of its ascent the velocity of the ball in the y-direction will be 0 m/s. The time taken to reach that maximum height can be found from the equation,

\[ t = \frac{v_{2y} - v_{1y}}{a} \]

Solving for \( t \) gives,

\[ t = \frac{v_{2y} - v_{1y}}{a} \]

where \( a = -9.8 \text{ m/s}^2 \)
\[ v_{1y} = v_1 \sin 60° \]
\[ v_{2y} = 0 \text{ m/s} \]

Therefore,

\[ t = \frac{0 \text{ m/s} - v_1 \sin 60°}{-9.8 \text{ m/s}^2} \]
\[ t = \frac{0 \text{ m/s} - (2.0 \text{ m/s})(\sin 60°)}{-9.8 \text{ m/s}^2} \]
\[ t = 0.18 \text{s} \]

Since the ball traces out a parabolic path, doubling this time will allow us to figure out how far the ball...
travels in the x-direction (i.e. the range). This distance is how far the hands should be apart and is given by:

\[ d_x = v_x t + \frac{1}{2} a_x t^2 \]

where acceleration in the horizontal direction is zero (i.e., \( a_x = 0 \) m/s\(^2\)). Thus,

\[ d_x = v_x t \\
\]

\[ d_x = v_x \cos 60^\circ t \\
\]

\[ d_x = (2.0 \text{ m/s})(\cos 60^\circ)(2 \times 0.18 \text{ s}) \\
\]

\[ d_x = 0.36 \text{ m} \]

We can also figure out how high the ball will travel using the following equation:

\[ d_y = v_y t + \frac{1}{2} a_y t^2 \]

\[ d_y = (2.0 \text{ m/s})(\sin 60^\circ)(0.18 \text{ s}) + \frac{1}{2} (-9.8 \text{ m/s}^2)(0.18 \text{ s})^2 \]

\[ d_y = 0.15 \text{ m} \]

These calculations reveal that a ball initially travelling at 2.0 m/s will reach a height of 0.15 m in 0.18 s, and that horizontally it will travel 0.36 m. This information will be extremely useful to a juggler (especially a novice juggler).

To juggle two balls under these conditions would entail throwing the second ball just as the first is reaching its peak (at around 0.18 s). For three balls, you would start with two balls in one hand and one in the other. The third ball would then be thrown when the second is at its peak and the first ball again when the third reaches its peak. The trick is to throw the third ball and catch the first one with the same hand, in the limited time available.

**Conclusion**

A basketball or volleyball coach would probably advise his/her players to keep his/her eye on the ball. The best advice for a juggler, however, would be to keep his/her eye off the ball, since a jugglers' attention must continually shift from one ball to the next. Possessing information on the timing and height of ball flight would make this job a little easier. Becoming a good juggler then requires patience, practice and physics.

**Questions**

1. A ball is thrown at an initial velocity of 3.0 m/s upward at an angle of 80° to the horizontal. How high will the ball rise? How far apart should the juggler hold his/her hands?

2. A juggler throws a ball at an angle of 70° to the horizontal. If the ball took 0.20 s to reach its maximum height, at what initial velocity was it thrown?

3. How is knowledge of projectile motion principles useful to a juggler?

4. Research: Who holds the current world record for greatest number of balls juggled?

**References**


